Field Inhomogeneity Correction based on Gridding Reconstruction

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Abstract

A new algorithm for the compensation of off-resonance effects is proposed. It is based on the same approximation as gridding reconstruction, and it is compatible with direct and iterative approaches. It achieves a similar accuracy as least squares methods, while requiring no time-consuming calculation of interpolation coefficients and offering further advantages in terms of computational complexity. Moreover, it permits to balance the accuracy of reconstruction and correction for non-Cartesian acquisitions. The algorithm is demonstrated in simulations and phantom experiments.

Introduction

Main field inhomogeneity distorts the Fourier encoding used to spatially resolve the detected signal and gives rise to image artifacts if disregarded in reconstruction. Several algorithms have been developed in the past to correct it [1-3]. Especially those for non-Cartesian acquisitions generally include an interpolation to reduce their computational complexity to an acceptable level. So far, this interpolation has been introduced independent of the approximation that the reconstruction of such acquisitions usually relies on. The present work proposes a unified approach.

Methods

Gridding and non-equispaced Fast Fourier Transform (NFFT) algorithms [4] employ an approximation of the form

$$e^{ikx} \approx \frac{1}{\alpha N \hat{w}(x)} \sum_{l=-\alpha N/2}^{\alpha N/2-1} w'\left(k - \frac{l}{\alpha N}\right) e^{ixl/\alpha N},$$

where α denotes an oversampling factor, N the number of samples in the image domain, w' a window function of kernel size $2m$, w the periodization of w', and $\hat{w}$ the Fourier transform of w. While originally holding for $x = -N/2, \ldots, N/2-1$ and $k \in [-\pi, \pi]$, it can be shown that this approximation remains valid for real $x \in [-N/2, N/2]$ if $k \in [-\pi+2\pi m/\alpha N, \pi-2\pi m/\alpha N]$.

Most field inhomogeneity correction algorithms for non-Cartesian acquisitions, including both direct and iterative ones, involve the computation of

$$s_\kappa = \sum_{\rho=0}^{N_1 N_2 - 1} m_\rho e^{\mathrm{i}\omega_\rho t_\kappa} e^{\mathrm{i}\mathbf{k}_\kappa \mathbf{r}_\rho}$$

or

$$m_\rho \approx \sum_{\kappa=0}^{M-1} d_\kappa s_\kappa e^{-\mathrm{i}\omega_\rho t_\kappa} e^{-\mathrm{i}\mathbf{k}_\kappa \mathbf{r}_\rho},$$

where s_κ denotes the estimated signal at position $\mathbf{k}_\kappa$ at time t_κ, m_ρ the magnetization and ω_ρ the angular off-resonance frequency at position $\mathbf{r}_\rho$, $N_1 N_2$ the number of pixels, M the number of samples in the k-space domain, and d_κ an optional sampling density compensation. In principle, ω and t may be considered as additional dimensions in image and k-space, respectively. Evaluating one of the sums then amounts to calculating a trivariate Fourier transform with non-equispaced sampling in both domains. To improve accuracy and efficiency, we suggest instead to apply the above approximation directly to the exponential function describing the influence of the field inhomogeneity. For this purpose, we define

$$N_3 \geq \frac{2\max(|\omega_\rho t_\kappa|)}{\pi} + \frac{2m}{\alpha}$$

and

$$T = \frac{\max(|t_\kappa|)}{\pi(1 - 2m/\alpha N_3)}.$$

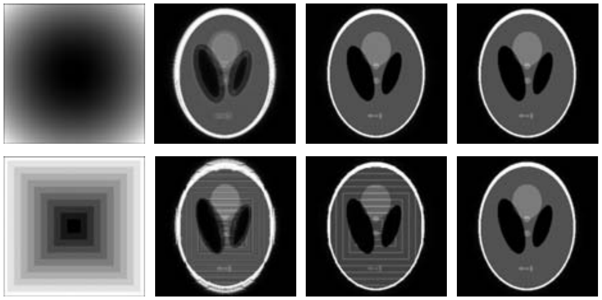

Fig. 1: Results of simulations. Spiral k-space data were calculated for a readout duration of 32 ms and the two field maps in the first column with off-resonance frequencies in the range of –125 Hz and +125 Hz. Standard gridding reconstruction produced the images in the second column, and the proposed correction algorithm, integrated into a conjugate phase and an iterative reconstruction, those in the third and fourth column, respectively. Two iterations were used in case of the continuous field map, and ten iterations in case of the discrete field map.

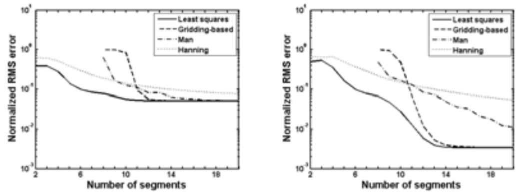

Fig. 2. Comparison of accuracy. The normalized root mean square (RMS) error is plotted as function of the number of segments in the interpolation. The left and the right graph show results of the conjugate phase and the iterative reconstruction, respectively, using the data for the continuous field map from Fig. 1. Least-squares [3], Man [2], and Hanning [1] denote existing correction algorithms, Gridding-based the proposed new one.

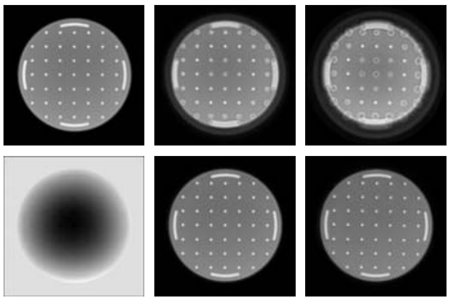

Fig. 3. Results of phantom experiments. A reference image and a field map obtained with Cartesian k-space sampling are presented on the left, and two corresponding uncorrected and corrected images acquired with spiral k-space sampling in the middle and on the right. The readout duration was 28.5 ms and 56.5 ms, respectively.

Algorithm	Running time
Least squares	1530 ms
Gridding-based	840 ms
Man	1530 ms
Hanning	710 ms

Tab. 1. Measured running times of different correction algorithms per iteration, using comparable parameter settings.

The first of the two sums may then be rewritten as

$$s_\kappa \approx \sum_{l=-\alpha N_3/2}^{\alpha N_3/2-1} w'\left(\frac{t_\kappa}{T} - \frac{l}{\alpha N_3}\right) \sum_{\rho=0}^{N_1 N_2 - 1} \left(\frac{m_\rho e^{\mathrm{i}\omega_\rho T l/\alpha N_3}}{\alpha N_3 \hat{w}(\omega_\rho T)}\right) e^{\mathrm{i}\mathbf{k}_\kappa \mathbf{r}_\rho}.$$

Thus, its computation requires αN_3 times a weighting of the magnetization m_ρ and the calculation of a 2D NFFT, and once a local convolution along the t axis. The second sum may be expressed similarly.

To assess the resulting algorithm, we incorporated it into a conjugate phase and an iterative reconstruction [1,3]. We then applied it to simulated and measured spiral k-space data, using the same Kaiser-Bessel window for $\hat{w}$ [5] and identical settings for α and m for reconstruction and correction.

Results

Representative simulation results are summarized in Fig. 1. The proposed correction algorithm obviously provides a good approximation. Its accuracy is compared to that of others in Fig. 2 for $\alpha = 1.25$ and $m = 2$. The minimum αN_3 is 14 in this example. If more than 14 segments are used, the accuracy of the least squares and the gridding-based correction algorithms is dominated by that of the reconstruction. Running times of various algorithms for a 256 x 256 image matrix are listed in Tab. 1. Differences mainly result from the varying locality of the convolution along the t axis. Finally, Fig. 3 demonstrates the gridding-based correction algorithm on phantom data, for which the resonance frequency deviated by ±95Hz.

Conclusions

For standard oversampling factors α and kernel sizes m, our gridding-based approach achieves a similar accuracy as least squares methods. It provides a rule for choosing the number of segments N_3 in the interpolation and allows a balance between the accuracy of reconstruction and correction. Additionally, it reduces the computational complexity, since the interpolation coefficients are simply given by the window function w' and the convolution along the t axis is local.

References

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